

factorial-positive

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0.1 Factorial

Definition 1 (Factorial). For any natural number n , we define the factorial of n , denoted by $n!$, recursively as:

- $0! = 1$
- $(n + 1)! = (n + 1) \cdot n!$

Theorem 2 (Positivity of Factorials). *For every natural number n , the factorial of n is strictly positive:*

$$n! > 0$$

Proof. By induction on n .

- Base case: $0! = 1 > 0$.
- Inductive step: Assume $n! > 0$. Then $(n + 1)! = (n + 1) \cdot n!$. Since $n + 1 > 0$ and $n! > 0$ (by inductive hypothesis), their product is positive.

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